# Chapter 8 Radiometry

Physically - Based Rendering Introduction.

## I. Photons.

1. In PBR, we are actually assuming that we are dealing with Photons.
2. In Graphics. we define photon as

Def (Photon) An energy packet that travels in straight lines, does NOT have wave properties.
It has a position, direction and wave length $\lambda$.

3. In practical. it is impossible to model micro-scale brute force renderer for photons.

Instead, we delve into bulk behavior. and this is called radiometry.

## II. Spectral Energy [J/nm] & Radiant Energy. [J]

remember $\lambda \Leftrightarrow$ color

1. Def (Photon Energy) $Q_i = \frac{hc}{\lambda}$, where $h$ is the Plank's Constant. c is the speed of light and $\lambda$ wave length.
2. Def (Spectral Energy) $Q_\lambda = \frac{\Delta Q}{\Delta \lambda}$. it is the energy on a certain $\lambda$ - interval.
3. Def (Radiant Energy) $Q = \sum_{i \in \Gamma} Q_i$ it is the total energy over all $\lambda$.

Intuitively. radient energy is the total # of hits of photons.

## III. Spectral Flux [J/nm·s] = [W/nm] & Radiant Flux. [J/s] = [W]

1. Def (Spetral Flux / power) $\Phi_\lambda = \frac{dQ_\lambda}{dt}$
2. Def ( Radiant Flux/ power) $\Phi = \frac{dQ}{dt}$ $\Rightarrow$ i.e. $Q = \int_{t_0}^{t_1} \Phi(t)\, dt$.

Intuitively, radient power, or flux is the # of hits per second.

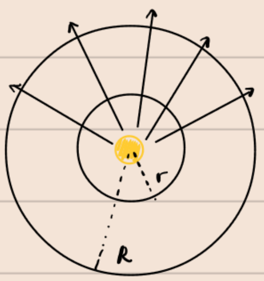
## IV. Irradiance. [W/m²]

1. Def (Irradiance) $E(\vec{p}) = \frac{d\Phi(\vec{p})}{dA}$, $\vec{p}$ is a point.

i.e. How much light hits the area in a sec ?

Intuitively, irradiance measures the area density of flux over the lighted surface.

2. Ex (Point Light). Consider the following point light:




The energy is uniformly distributed over the spherical area.

Thus. for inner surface. $E = \frac{\Phi}{4\pi r^2}$.

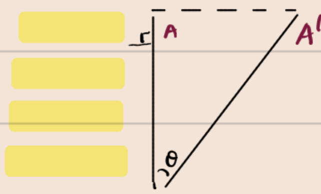
for outer surface. $E = \frac{\Phi}{4\pi R^2}$

flux the same.

The light falls off with the squared distance.

3. Lambert's Law.

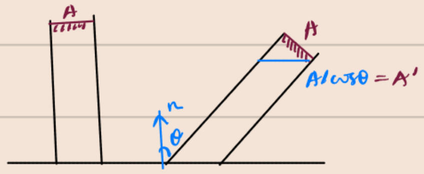
Intuition



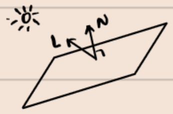

When light beam form an angle $\neq 90°$ with the surface. (A') (A)

the equivalent irradiance should be the projected area.

Thm ( Lambert's Law ) $E = \frac{\Phi}{A'} = \frac{\Phi \cos\theta}{A}$



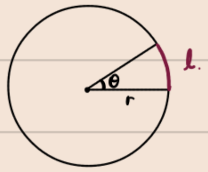

Ex ( NdotL lighting ). Simply return $\max(0, \hat{N} \cdot \hat{L})$;




## V. Radiance. $[W/m^2 \cdot sr]$

1. Def ( Angle ) The ratio of subtended arc length to radius.

$\theta = l/r$ or $l = r\theta$

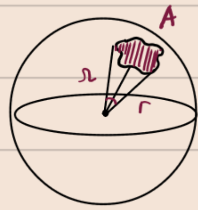



(circle)

Thus. $360° \Leftrightarrow 2\pi r/r = 2\pi$. ( radians ).

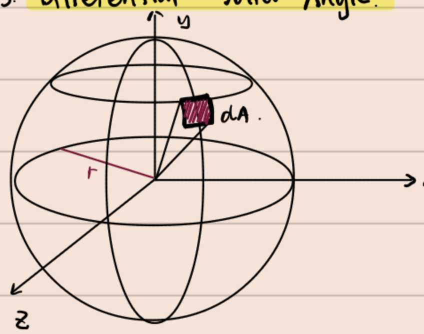
2. Def ( Solid Angle ). The ratio of subtended area on sphere to radius squared.

$\Omega = A/r^2$ or $A = \Omega r^2$

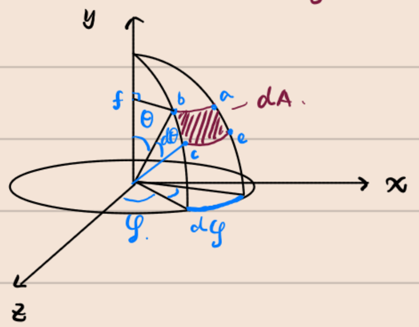



Thus, sphere has $4\pi r^2/r^2 = 4\pi$ ( steradians )

3. Differential Solid Angle. used for describe the solid angle on a single direction.




$\Rightarrow$




$d\theta, d\varphi \to 0$. $d\omega = \frac{dA}{r^2}$

$dA = \|ab\| \cdot \|cb\|$.

① $\|bc\| = r\, d\theta$. ( $l = r\theta$ ).

② $\|bf\| = r\sin\theta$ ( trigometry ).

$\Rightarrow \|ab\| = r\sin\theta\, d\varphi$. ( $l = r\theta$ ).

Thus. $dA = r\,d\theta\; r\sin\theta\, d\varphi$.

$d\omega = r^2 \sin\theta\, d\theta\, d\varphi / r^2$

$= \sin\theta\, d\theta\, d\varphi$.

And indeed.

$\Omega_{sphere} = \int_{S^2} d\omega$

$= \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\varphi = 4\pi$.

4. Now, we will use $\hat{\omega}$ as a direction vector. with unit length.

5. Def ( Radiance ). The solid angle density of irradiance.

$L(p,\omega) = \lim_{\Delta\omega \to 0} \frac{\Delta E_\omega(p)}{\Delta\omega \cos\theta} = \frac{dE_\omega(p)}{d\omega\cos\theta}$ i.e. $E = \int_{H^2} L(\omega)\cos\theta\, d\omega$.

Intuitively. radiance is energy along a ray defined by origin point $\vec{p}$ and direction $\hat{\omega}$.

i.e. radiance doesn't change for a light source for a given direction.

6. Not that in general. incident radiance $L_i(\vec{p}, \hat{\omega}) \neq$ exitant radiance $L_o(\vec{p}, \omega)$.

# VI. BRDF.

1. Def (BRDF) The bidirectional reflectance distribution function. is called the BRDF.

   Input : Incoming direction $\hat{w}_i$, Exitting direction $\hat{w}_o$. ( denoted as $f_r(\hat{w}_i \to \hat{w}_o)$)

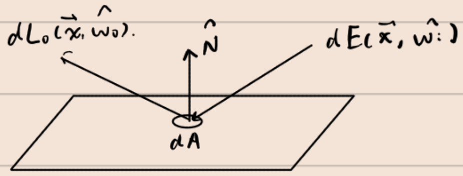
   Output . How much light has been scattered ?

   Properties : ① $f_r(\hat{w}_i \to \hat{w}_o) \geq 0 \quad \forall\ i, o.$

   ② $\int_{H^2} f_r(\hat{w}_i \to \hat{w}_o) \cos\theta \, dw_i \leq 1.$

   ③ $f_r(w_i \to w_o) = f_r(w_o \to w_i).$

$$f_r(\hat{w}_i \to \hat{w}_o) = \frac{dL_o(\hat{w}_o)}{dE_i(\hat{w}_i)} = \frac{dL_o(\hat{w}_o)}{dL_i(w_i)\cos\theta_i} \quad \left[\frac{1}{sr}\right]$$




2. Intuitively. BRDF is a 4D function describing reflectance that
   - returns a ratio of surface radiance of exitance direction. by an incident irradiance of incident direction.
   - takes directions.

# VII. Rendering Equations.

1. The Rendering Equation Rendering is all about calculating radiance.

① $L_o(\vec{p}, \hat{w}_o)$ : query for (observed) outgoing radiance for query point $\vec{p}$ and direction $\hat{w}_o$.

② $L_e(\vec{p}, \hat{w}_o)$ : emitted radiance if this is a light source.

can be other models

③ $\int_{H^2} f_r(\vec{p}, \hat{w}_i \to \hat{w}_o) L_i(\vec{p}, \hat{w}_i) \cos\theta \, dw_i$ : integral on all directions in hemisphere for BRDF × incoming radiance × cos of angle b/t incoming direction and normal.

$$L_o(\vec{p}, \hat{w}_o) = L_e(\vec{p}, \hat{w}_o) + \underbrace{\int_{H^2}}_{\text{hemisphere}} f_r(\vec{p}, \hat{w}_i \to \hat{w}_o) L_i(\vec{p}, \hat{w}_i) \cos\theta \, dw_i.$$

this requires another integral.

Therefore. the rendering equation is recursive.

2. Renderer measures radiance along a ray.

3. BRDF accounts for how the reflection of light affects the outgoing radiance.

4. By shorthand, we can write

$$L = E + KL = E + KE + K^2E + K^3E + \cdots$$

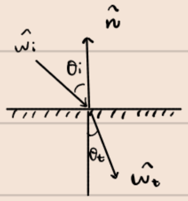
## VIII. Scattering.

For a photon. besides being reflected or transmitted through the surface. it may also

- bounces around inside the surface.
- absorbed and re-emitted.

} these are called Scattering.

### 1. Snell's Law.

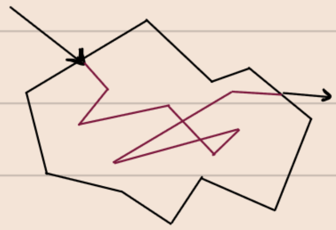



**Def** (Snell's Law) $\eta_i \sin\theta_i = \eta_t \sin\theta_t$.

$\eta$ is the refraction factor.

### 2. Subsurface Scattering (SS).

Some times light exits @ a different point than it enters.



We generalize BRDF to get a more descriptive BSSRDF.

**Def** (BSSRDF) Returns the ratio, given $\vec{p_i}$, $\hat{w_i}$, $\vec{p_o}$, $\hat{w_o}$

incident point, incident dir., exitant point, exitant dir.

**Def** (Render Equation, SS) $L(\vec{x_o}, \hat{w_o}) = \int_A \int_{H^2} S(\vec{x_i}, \hat{w_i}, \vec{x_o}, \hat{w_o}) L_i(x_i, w_i) \cos\theta_i \, dw_i \, dA$.

### 3. The Reflection Equation.

Scattering is very complicate. we can use the reflection equation to simply the Render Equation.

① Approximate integral via Monte Carlo Integration. (will discuss later).

② Generate directions $\hat{w}_j$ sampled from some distribution $p(w)$.

③ Compute Estimator. $\frac{1}{N} \sum_{j=1}^{N} \frac{f_r(p, w_j \to w_o) L_i(p, w_j) \cos\theta_j}{p(w_j)}$.